Utility Economic Analysis using Interval Mathematics

by
Justinn Kase, Utility Engineer
Independent Regulatory Commission - IRC

Introduction

- Making informed decisions involving money is a need of the utility industry. Thus, alternatives must be considered. Alternatives may involve the construction of a new generation plant, the replacement of aging equipment and the like. The costs associated with these alternatives occur at different times in the project.

Introduction

- The revenue requirement method is frequently utilized by utilities for the analysis of alternatives involving money. These analyses frequently rely on data containing significant levels of uncertainty. In the past, it was difficult to assess the impact of these uncertainties on the final outcome of the economic study.

Introduction

- Interval analysis provides a method of including these uncertainties within the analysis process and presenting the solution in the form of interval bounds inclusive of all possible solutions.

Revenue Requirement Approach

- The Revenue Requirement Method takes into consideration all costs associated with each project alternative and these costs are reduced to the revenue required from the utility's customers to support that alternative.
- The required revenue is equal to the costs associated with the initial placement of the plant plus the operating expenses. This approach provides the most attractive economic choice and the impact that that choice will have on the utility's customers.

Revenue Requirement Approach

- The data employed in performing the economic analysis comes from many different sources and possess varying degrees of accuracy. It can be rather challenging to fully allow for the effects that such uncertainties would have on the final outcome of an economic study.

Revenue Requirement Method

- The Revenue Requirement consists of two major components:
  1. Original capital cost of the project, and
  2. Operating expense to include salaries, maintenance and fuel costs.

  The annual sum of the costs associated with the original placement of plant is called the ”*carrying charge*”. Thus, the total revenue requirement for a given year will be the sum of the carrying charges and expenses.

Approaches to Economic Analysis

- There are several accepted methods to approaching utility economic analysis. The five common approaches are:
  1. Book Life Analysis
  2. Year by Year Analysis
  3. Continuing Plant Analysis
  4. Short Term Analysis
  5. Break Even Analysis

Interval Mathematics

- Interval mathematics takes into consideration the uncertainty associated with the parameters used in a computation. It uses interval “numbers” which are an ordered pair of real numbers representing the lower and upper bound of the parameter range.

  Example: If the interest rate on a loan lies between 8% and 12% it would be represented as $i = [0.08, 0.12]$

Fundamental Axioms of Interval Mathematics

- The rules of addition, subtraction, multiplication and division are shown below:

  [a,b] + [c,d] = [a+c, b+d]

  [a,b]-[c,d] = [a-d,b-c]

  [a,b]*[c,d] = [min(a*c,a*d,b*c,b*d), max(a*c,a*d,b*c,b*d)]

  [a,b]/[c,d] = [a,b]*[1/d,1/c], where 0 not an element of [c,d]

Commutative and Associative Properties

If X=[a,b], Y=[c,d] and Z=[e,f], then

X+(Y+Z)=(X+Y)+Z

X*(Y*Z)=(X*Y)*Z

X+Y=Y+X and X*Y = Y*X

The distributive law also gives:

X*(Y+Z)=X*Y+X*Z and is true only when XY > 0.

Further information on Interval Mathematics can be found at http://en.wikipedia.org/wiki/Interval_(mathematics)

Application to Break Even Analysis

- Many parameters used in utility economic analysis are not absolutely precise. For example: The cost of constructing a facility will be known as a range of costs. Operating expenses, inflation rates and the cost of money are all subject to some degree of uncertainty.

Break Even Analysis Example

- Algorithms were developed according to user defined functions which in turn perform mathematical operations according to interval algebraic rules.
- Interval mathematics will now be used for a representative utility break even analysis problem.
- The analysis will show the results for a single point parameter as well as for interval parameters which takes into consideration an assumed range of uncertainty in the data.

Single Point Parameter Example

- The utility is presented with two competing alternatives involving power generation. Both alternatives have a book value of 30 years.
- Alternative A – Initial capital costs of $1.5M and operating costs of $65,000 per year.
- Alternative B – Represents continued operation of the present generation system. Initial cost =$0 and operating costs =$200k per year.

Single Point Parameter Example Continued

- Additional economic parameters include:
- Effective Interest Rate – 10%/year
- Cost of Debt – 8%
- Cost of Preferred Shares – 8.5%
- Cost of Common Shares – 13.5%
- Apparent Escalation = 7.696% (Based on 6% inflation and 1.6% real escalation)
- Property Taxes and Insurance = 2%
- Income Tax Rate = 38% and Investment tax Credit = 0

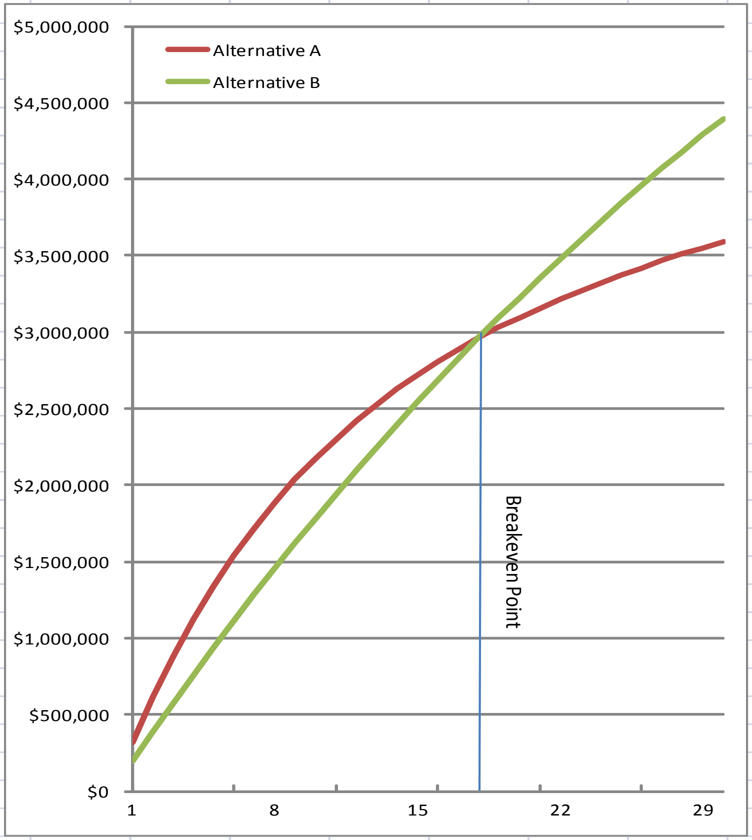
Single Point Parameter Example Continued

- The depreciation method to be used is straight line over book life. It was assumed , initially, the values of all variables are known exactly.
- The summary of the revenue requirements for investments A and B are shown in Table 1. which represents the cumulative present value of the yearly costs associated with Plans A and B.

Single Point Parameter Example Continued

Table 1. Cumulative Present Worth of Revenue Requirements for Alternatives A and B
Non-Interval Case

Year	Alternative A in Dollars	Alternative B in Dollars
1	$322,876	$195,811
2	615,200	387,520
3	880,171	575,215
4	1,120,650	758,977
5	1,339,199	938,891
6	1,538,111	1,115,037
7	1,719,436	1,287,493
8	1,885,008	1,456,336
9	2,036,467	1,621,644
10	2,175,279	1,783,488
11	2,302,755	1,941,943
12	2,420,066	2,097,079
13	2,528,259	2,248,966
14	2,628,269	2,397,671
15	2,720,932	2,543,262
16	2,806,993	2,685,803
17	2,887,118	2,825,359
18	2,961,902	2,961,991
19	3,031,876	3,095,762
20	3,097,514	3,226,731
21	3,159,239	3,354,956
22	3,217,428	3,480,496
23	3,272,418	3,603,406
24	3,324,510	3,723,742
25	3,373,971	3,841,558
26	3,421,040	3,956,905
27	3,465,932	4,069,837
28	3,508,836	4,180,403
29	3,549,923	4,288,654
30	3,589,344	4,394,637



Single Point Parameter Example Continued

Figure 1 depicts the cumulative present values of the yearly costs associated with Plans A and B. Shown is that the cumulative present values of the costs for the two alternatives are equal at approximately year 18. This would be the break even point for the two alternatives.

Interval Parameter Example

- Consider a few likely uncertainties in the same data given in the previous example:
- Effective interest rate = [9.75, 10.25]%,
- Cost of Debt = [7.75, 8.25]%,
- Cost of Preferred Shares = [8.25, 8.75]%,
- Cost of Common Shares = [13.25, 13.75]%

Interval Parameter Example Continued

For the two alternatives given above we have:

- Alternative A: Initial Costs = [$1.4M,$1.6M] Operating Costs = [$60K, $70k]/year
- Alternative B: Operating Costs = [$190K, $120K]/year

Uncertainties in both the initial cost of Alternative A and the operating costs have been added.

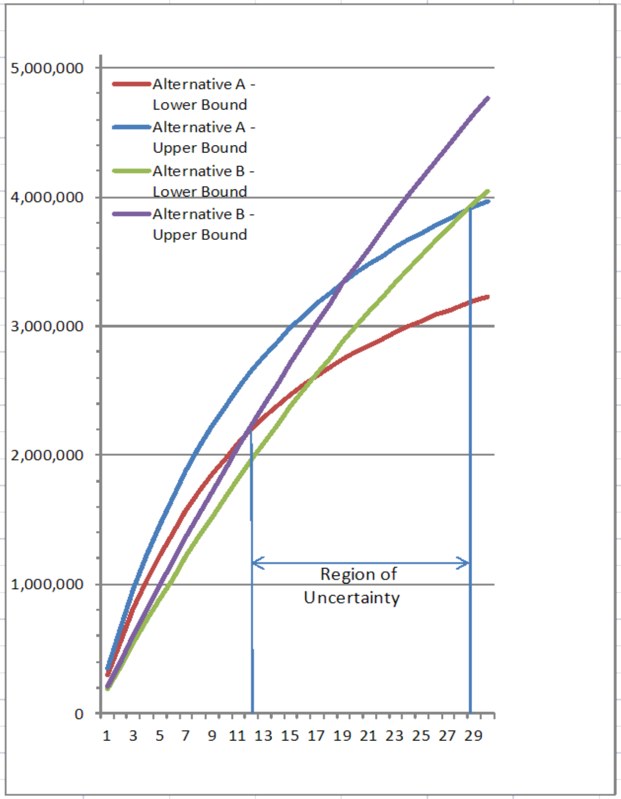
Interval Parameter Example Continued

- Table 2 shows the summary of the revenue requirements for Alternatives A and B. Interval mathematics allow the construction and operating costs to vary within the stated limits. The revenue requirements are now in interval form.

Interval Parameter Example Continued

Table 2. Cumulative Present Worth of Revenue Requirements for Alternatives A and B in Dollars-Interval Case

Year	Alternative A Interval Bounds - $		Alternative B Interval Bounds - $	
1	295,869	350,601	185,599	206,070
2	563,196	668,672	366,898	408,283
3	805,018	957,563	543,997	606,712
4	1,024,043	1,220,279	716,993	801,427
5	1,222,695	1,459,516	885,982	992,498
6	1,403,133	1,677,693	1,051,057	1,179,992
7	1,567,289	1,876,975	1,212,307	1,363,978
8	1,716,883	2,059,306	1,369,822	1,544,521
9	1,853,452	2,226,426	1,523,688	1,721,685
10	1,978,368	2,379,894	1,673,989	1,895,533
11	2,092,854	2,521,108	1,820,809	2,066,127
12	2,198,001	2,651,319	1,964,227	2,233,529
13	2,294,781	2,771,646	2,104,324	2,397,797
14	2,384,063	2,883,093	2,241,174	2,558,992
15	2,466,619	2,986,557	2,374,855	2,717,169
16	2,543,139	3,082,842	2,505,439	2,872,387
17	2,614,238	3,172,664	2,632,997	3,024,699
18	2,680,463	3,256,668	2,757,601	3,174,161
19	2,742,301	3,335,427	2,879,318	3,330,825
20	2,800,189	3,409,456	2,998,216	3,464,745
21	2,854,513	3,479,213	3,114,359	3,605,971
22	2,905,619	3,545,110	3,227,812	3,744,554
23	2,953,814	3,607,513	3,338,637	3,880,544
24	2,999,372	3,666,750	3,446,894	4,013,988
25	3,042,537	3,723,114	3,552,643	4,144,935
26	3,083,528	3,776,867	3,655,943	4,273,431
27	3,122,538	3,828,242	3,756,850	4,399,523
28	3,159,740	3,877,448	3,855,419	4,523,254
29	3,195,289	3,924,672	3,951,704	4,644,670
30	3,229,321	3,970,081	4,045,760	4,763,814



Interval Parameter Example Continued

Figure 3, i.e. the graph preceding this slide depicts the cumulative present values of costs associated with Alternatives A and B.

Due to the uncertainty in the initial and operating costs, the breakeven point is less well defined and consists of the interval between 11 and 28 years. This indicates that the breakeven point is sensitive to variations in the construction and operating costs.

Drawbacks of Interval Analysis

- Interval Mathematical techniques have several drawbacks due to its arithmetic which does not conform to the normal rules of arithmetic. This can cause larger than expected bounds in any solution.
- The lack of easily implemented algorithms have hampered its widespread use. However, considerable progress has been achieved over the past few years in correcting this problem.

Conclusions

- Interval Mathematics can provide significant insights into the effects of parameter uncertainties on the results of utility economic analysis.
- Interval analysis carries out sensitivity analysis concurrently with normal computations and does not rely on either a priori or aposteriori analysis.
- Interval analysis allows any or all parameters to vary simultaneously in order that the impact of all variations can be easily studied.
- The final results can be presented in the form of bounds which contain for all practical purposes all possible solutions.

References

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Questions

Thank you

Thank you for your kind attention!